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EMPIRICAL FORMULA FOR THICK TARGET PARTICLE PRODUCTION

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I. Introduction

A simple analytical expression for particle production from thick targets is of great practical importance. One interaction length solid targets are commonly used at Fermilab for many of the secondary beamlines. An accurate analytic formula that predicts the momentum and angular distributions of produced particles is helpful in designing beamlines, and performing flux calculations that use computer programs like NUADA[1], DECAY TURTLE[2], HALO[3], and others. This is particularly true for beams having large momentum acceptances, such as neutrino horn beams, dichromatic beams, and muon beams.

The formula is based on a fit utilizing the data of Atherton et. al.[4], who made precise measurements of particle production by 400 GeV/c protons on beryllium targets. The formula contains four free parameters for each particle type. Fits for the 500 mm target data are obtained using MINUIT[5]. The fits agree well with the data, giving $\chi^2/DF \simeq 1.$ for 10% errors on the data.

II. Variables and Parameters

The laboratory differential cross section is

$$\frac{d^2N}{dP\,d\Omega} = \frac{P^2}{E\,\sigma}\,I \simeq \frac{P}{\sigma}\,I \qquad \text{for high energies.}$$

I is the invariant cross section.

P is the laboratory momentum of the produced particle.

E is the energy of the produced particle.

σ is the total inelastic cross section.

The Atherton data is presented as $\dfrac{d^2N}{\frac{dP}{P}d\Omega} = \dfrac{P^2}{\sigma}I = \dfrac{E^2}{\sigma}X^2 I$

X is a scaling variable taken to be the momentum of the produced particle divided by the energy of the primary incident protons.

The invariant cross section is represented by a factorized scaling form in X and P_t; $I = f(X)g(P_t)$.[6] The function f(X) is taken as $(1.-X)^A$ [7], and $g(P_t)$ is taken as $(1.+P_t^2/M^2)^{-4}$. It is known that the invariant cross sections exhibit a power law dependence $1/P_t^8$ at large P_t. [8] The factor $(1.+P_t^2/M^2)^{-4}$ is used with M^2 as a free parameter to account for the observed variation from $1/P_t^8$ behavior at low P_t.

Since the results are for a thick target, a third term is included to empirically account for the nuclear cascade process, which leads to more particles at low X. The fit uses $(1.+5e^{-DX})$; D is positive for all particles except protons and gives the desired increase at small X. D is negative for protons giving rise to the fact that more and more protons are produced at high X. This term has the effect of increasing the number of protons and also counteracting the $(1.-X)^A$ term at high X.

A minimum χ^2 fit was performed on the data to determine the values of the parameters A,B,M^2,and D for each particle type:

$$\frac{d^2N}{\frac{dP}{P}d\Omega} = \text{constant } X^2 I = \text{constant } X^2\,\frac{(1-X)^A(1+5e^{-DX})}{(1+P_t^2/M^2)^4}$$

This is changed to a more standard form and the parameters are given in Table I.

$$\frac{d^2N}{dPd\Omega} = B\,X\,\frac{(1-X)^A(1+5e^{-DX})}{(1+P_t^2/M^2)^4}$$

(Number of produced secondaries per steradian-GeV/c-incident proton on a 500 mm beryllium target.) Table II compares the data with that calculated from the fit.

III. Discussion

A. The Formula

Excluding proton production, the correction term $5e^{-DX}$ is small for most of the X range, because D is greater than 10. The major effect of this correction term comes at small X, and in fact, the motivation for choosing such a term comes from a low energy peak that is observed in the wide-band neutrino horn spectrum. Horn spectra comparing the thick target parameterization with that of Stefanski-White [9] are shown in Figure 1. They are taken from NUADA results with 400 GeV/c incident protons on the Mori Horn and use the MIDBIN calculation.

For protons, the term $5e^{-DX}$ becomes the dominant factor in $(1+5e^{-DX})$ because D is negative. However at large X, even this growing positive exponential is being brought to zero with the $(1-X)^A$ term, and therefore should not be used to predict proton production for $X > 0.75$. At that point, $(1-X)^A$ dominates and as X increases, the calculated number of protons actually decreases. Proton production was cast into the same form as the other particles merely as a convenience in computer programming. Since the data fits quite well up to X=0.75, there is no need to postulate a different formula. The majority of cases do not require calculations at higher X.

B. Adapting the Formula

In order to make this information more useful, guidelines are given to modify the empirical formula for other target lengths and other target materials. The size of errors that are to be expected from the modifications are also indicated.

a. Changing target length

In general, π and K production as a function of target length show small variations from the naive reabsorption model in which the produced secondaries are reabsorbed in the target without producing additional particles. For a target of length L, the model gives a target production efficiency f, described by:

$$f(L) = \frac{e^{-L/\lambda(s)} - e^{-L/\lambda(p)}}{1 - \lambda(p)/\lambda(s)}$$

where $\lambda(s)$ is the absorption cross section for the produced secondaries, and $\lambda(p)$ is the absorption cross section for protons. For secondary proton production, $\lambda(s)=\lambda(p)$, and the formula simply becomes:

$$f(L) = \frac{L}{\lambda(p)} e^{-L/\lambda(p)}$$

Absorption cross sections $\sigma(s)$ have been measured by Carroll et. al.[10] for several different materials. They are described by a power law A^{α} where A is the atomic weight and α is a constant obtained from Table I of reference 10. Values from lithium (A=7) can be adapted to beryllium. From Table I, $\lambda(p+)=0.72$, $\lambda(\pi\pm)=0.76$, and $\lambda(K\pm)=0.78$. The absorption lengths in centimeters is then,

$$\lambda(s) = \frac{A(gm)}{6.023} \; \frac{10^4}{\sigma(s) \quad (mb)} \; \frac{1}{\rho(gm/cm^3)}$$

For beryllium, $\lambda(p)=43.5$ cm, $\lambda(\pi\pm)=58.5$ cm, and $\lambda(K\pm)=65.7$ cm.

Thus, meson production for a beryllium target of length L is (f(L)/f(500 mm)) x (Empirical Formula). (Obviously L must be sensible and within bounds. Results are unpredicable if L is much greater than 500 mm.) In the example above, a 100 mm target would give (0.19/0.43) x (Empirical Formula).

The Atherton measurements also include data for 100 mm and 300 mm targets. If results obtained by multiplying the Empirical Formula by f(L)/0.43 are compared with the measurements for the short targets, it agrees within 15% except at one point. At X=0.5, P_t=0., the measured values (for π+ and K+ only) are $\simeq$ 30% lower than would be predicted by the above calculation. In gereral the other meson points show a variation of +15% (prediction higher than measured data) at X=0.15 to −15% (prediction lower than measured data) at X=0.75 .

b. Changing target material

As a rough guide, Figures 2 and 3 can be used to obtain rates for different target materials. The figures are based upon measurements made by Eichten et. al.[11] using 24 GeV/c primary protons. Using a beryllium target that was thin, the relative production rates for the same particle as a function of X are the same for the Eichten measurements as the Atherton measurements. The lines for the various target materials are to guide the eye only.

I wish to thank Jorge Morfin for his helpful suggestions in using MINUIT and also for profitable discussions on particle production.

Table I. Parameter values for $\frac{d^2N}{dPd\Omega} = B\,X\,\frac{(1-X)^A(1+5e^{-DX})}{(1+P_t^2/M^2)^4}$

Units are produced particles/ (St)(GeV/c)(Incident Proton) for a 500 mm beryllium target.

	π^+	π^-	K^+	K^-	P	$\overline{P}$
A	3.598	4.122	2.924	6.107	1.708	7.990
B	177.2	70.60	14.15	12.33	3.510	5.810
M^2	.7077	.8932	1.164	1.098	1.043	1.116
D	27.00	11.29	19.89	17.78	−4.314	14.25

Table II. The 500 mm Atherton data (top line) compared with the fit (bottom line) for $d^2N/(dP/P)d\Omega$. P and P_t are measured in GeV/c. 300 GeV/c positive production at zero degrees was not measured. Units are particles/(St)(%dP/P)(Incident Proton).

P	P_t	π^+	π^-	K^+	K^-	P	$\overline{P}$
60	0.0	8.87	6.70	1.01	.572	2.29	.224
		9.66	6.24	.992	.554	2.53	.227
60	0.5	2.94	2.19	.442	.241	1.06	.102
		2.88	2.33	.456	.244	1.07	.101
120	0.0	20.3	7.60	1.79	.534	13.7	.135
		17.7	6.83	1.82	.515	13.2	.129
120	0.3	11.3	4.11	1.43	.330	10.5	.084
		11.0	4.65	1.35	.376	9.50	.095
120	0.5	5.72	2.72	.940	.232	6.55	.067
		5.28	2.54	.835	.227	5.60	.058
200	0.0	14.2	4.23	1.81	.197	49.1	2.44E-2
		14.6	4.13	1.86	.179	47.5	2.29E-2
200	0.5	3.87	1.50	.760	.082	17.5	9.60E-3
		4.37	1.54	.856	.079	20.1	10.2E-3
300	0.0	--	.483	--	5.56E-3	---	2.03E-4
		--	.524	--	5.84E-3	--	2.02E-4
300	0.5	.849	.218	.269	2.62E-3	41.1	9.00E-5
		.811	.195	.254	2.57E-3	40.1	9.01E-5

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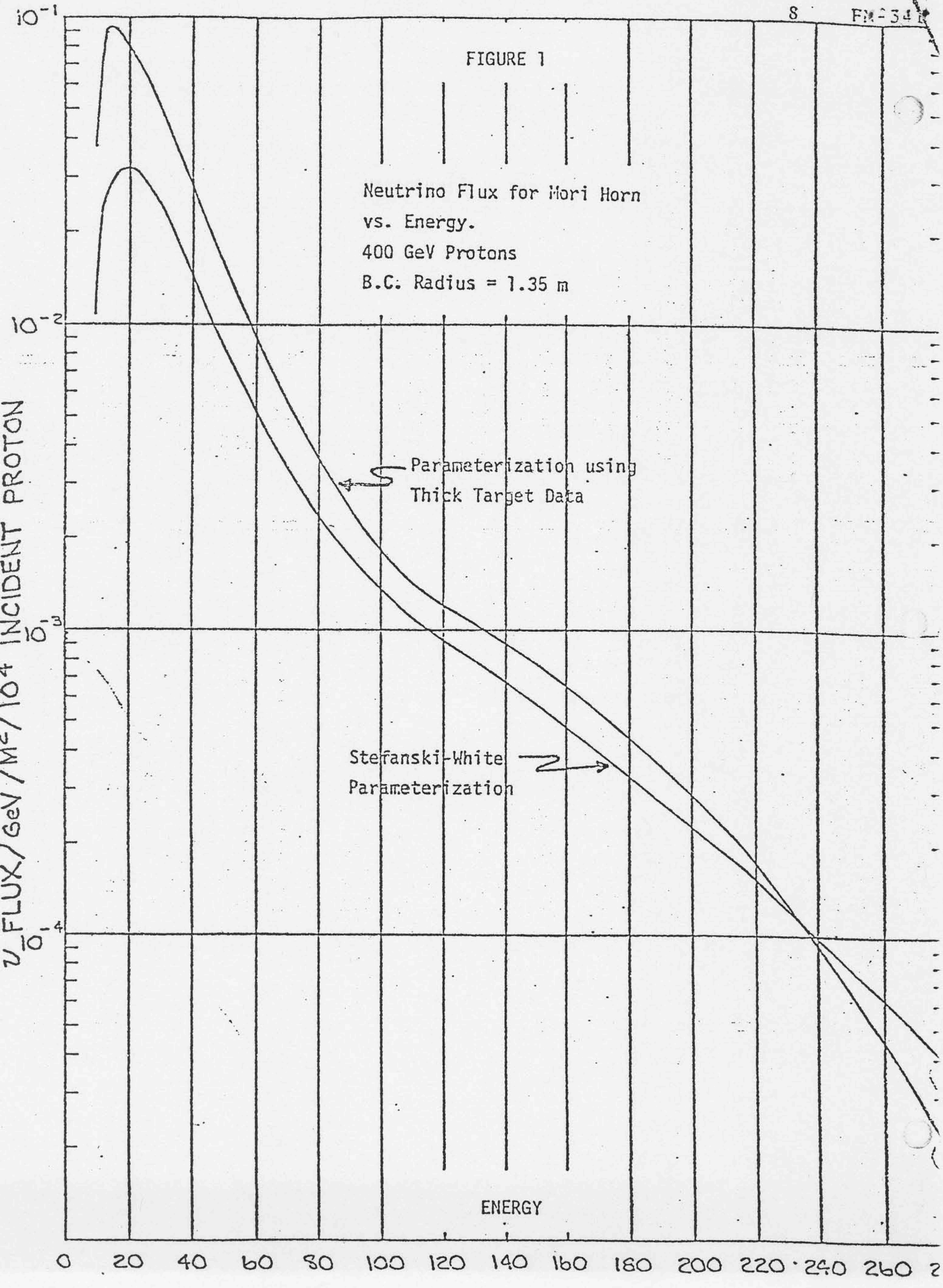


FIGURE 1
Neutrino Flux for Mori Horn
vs. Energy.
400 GeV Protons
B.C. Radius = 1.35 m
Parameterization using
Thick Target Data
Stefanski-White
Parameterization
ν FLUX/GeV/M^2/10^4 INCIDENT PROTON
10^{-1}
10^{-2}
10^{-3}
10^{-4}
ENERGY
0 20 40 60 80 100 120 140 160 180 200 220 240 260

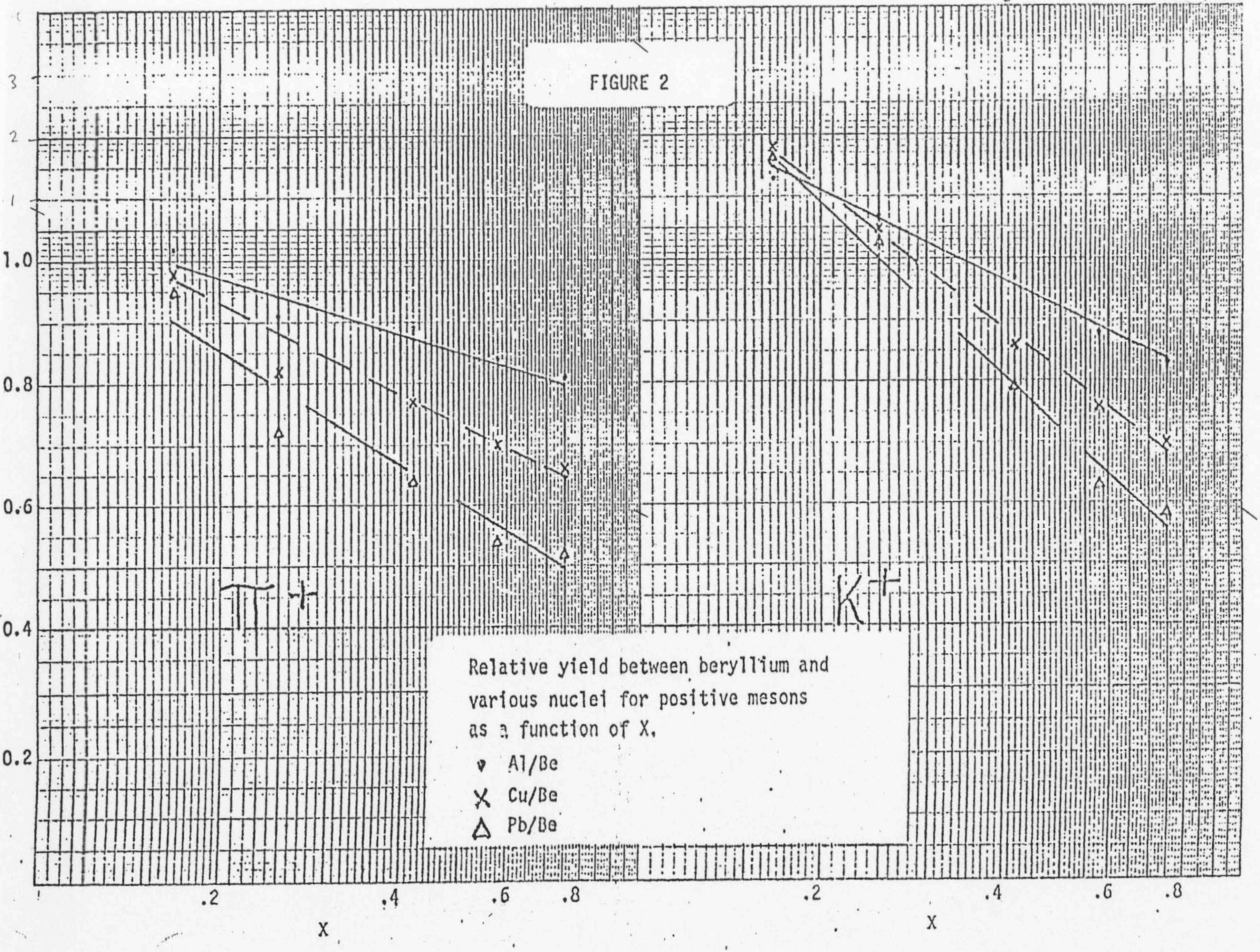
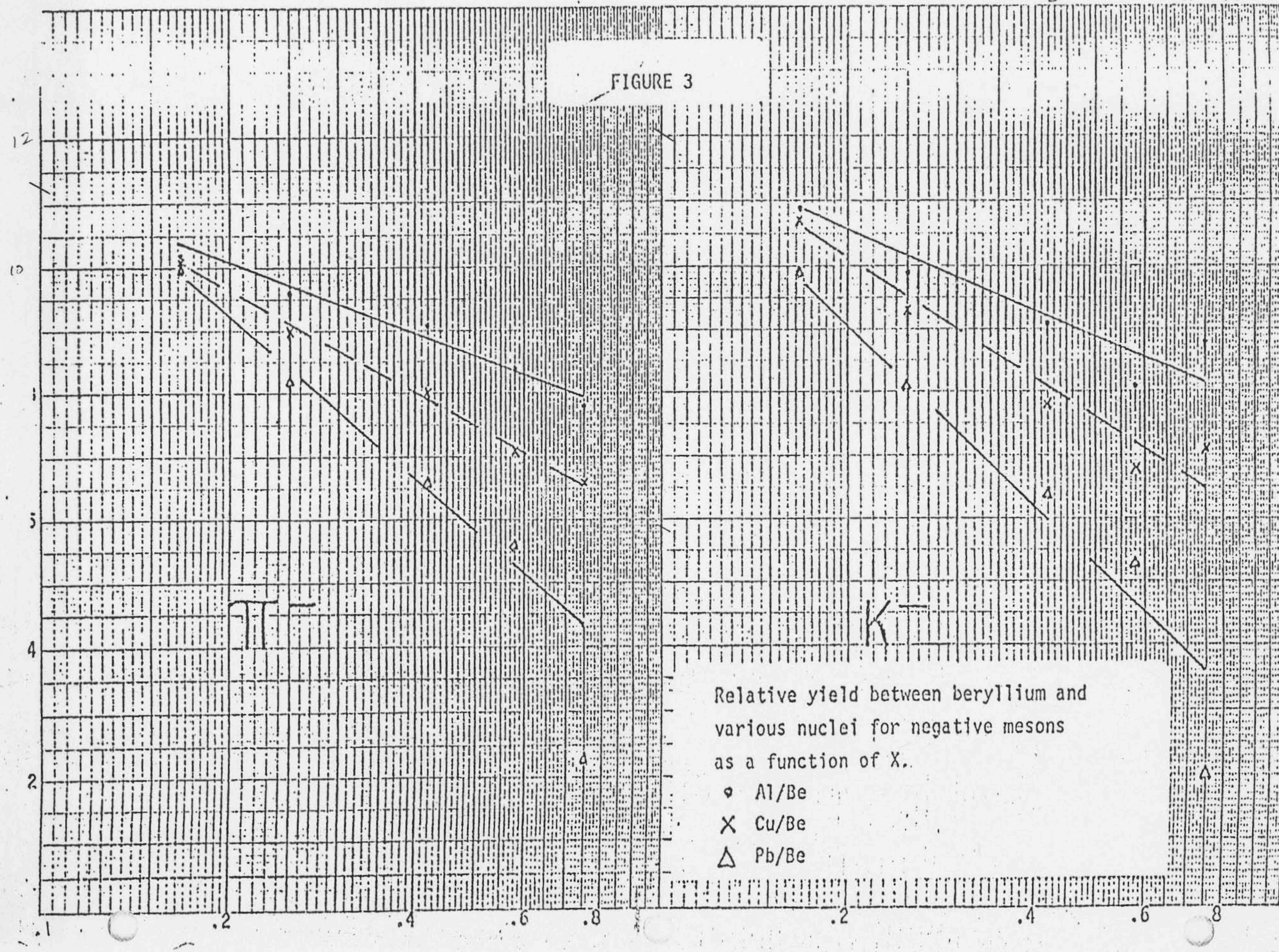


FIGURE 2

Relative yield between beryllium and various nuclei for positive mesons as a function of X.

FIGURE 3



Relative yield between beryllium and various nuclei for negative mesons as a function of X.

- ∘ Al/Be
- X Cu/Be
- △ Pb/Be

Addendum to
Empirical Formula for
Thick Target Particle Production

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The formula for $d^2N/dPd\Omega$ as given on page 3 and in Table I, is written specifically for 400 GeV protons on target. As contained in Table I, the constant B contains a factor E_0 (the energy of the primary proton beam). This factor must explicitly be taken out when one applies the formula at a different E_0.

Using a new constant K, we have

$$\frac{d^2N}{dPd\Omega} = K * P * I$$

$$\frac{d^2N}{dPd\Omega} = B * X * \frac{(1-X)^A * (1+5e^{-DX})}{(1 + P_t^2/M^2)^4} = B * X * I$$

Changing $X=P/E_0$ gives,

$$\frac{d^2N}{dPd\Omega} = B * \frac{P}{E_0} * I$$

The general formula is then,

$$\frac{d^2N}{dPd\Omega} = K * P * I = K * P * \frac{(1-X)^A * (1+5e^{-DX})}{(1 + P_t^2/M^2)^4}$$

With $K=B/400$. (B as given in Table I)