Partial-Wave Analysis and Baryon Spectroscopy in Pion-Nucleon Scattering

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Based on work in collaboration with
R. Arndt, W. Briscoe, R. Workman

- GW DAC N* Program
- SAID πN Analysis
- PW Amplitudes
- Resonances for S = 0 with I = 1/2 and 3/2
  Poles and BWs
- Summary

MIPP-FNAL Collaboration Meeting, Apr 28th, 2007
**N* and Δ* States Coupled to πN**

- One of the most convincing ways to study Spectroscopy of N* is πN PWA

- GW DAC SAID program: \( \pi N \rightarrow \pi N \Rightarrow \gamma N \rightarrow \pi N \Rightarrow \gamma^* N \rightarrow \pi N \)
- πN elastic amplitudes from fits to the observables: \( \sigma^{tot}, d\sigma/d\Omega, \) and \( P \) (plus a few R and A measurements, 0.4 %)
- Contain resonances contributing to \( \gamma N \rightarrow \pi N \)
- Assuming dominance of 2 hadronic channels, can parametrize \( \gamma N \rightarrow \pi N \) in terms of \( \pi N \rightarrow \pi N \) amplitudes alone
- Resulting multipoles can be re-fitted in terms of Res/bckgr contributions or used as input to multi-channel fits with more elaborate constraints
- A comparison of various resonance-extraction methods gives a more reliable estimate of systematic (model) errors
Objective

- Our PWAs have been as model-independent as possible, so as to avoid bias when used in resonance extraction or coupled-channel analysis.

- An example is provided by our elastic $\pi N$ analysis:
  - Resonances are found through a search for poles in the complex plane and are not put in by hand as BW terms.
  - This distinction is important for more complicated structures, such as the $N(1440)P_{11}$ and $N(1535)S_{11}$.
  - Also, it is an issue in search for `missed' or `hidden' resonances.
**GW SAID (Scattering Analysis Interactive Dial-in) Facility**

[http://gwdac.phys.gwu.edu/](http://gwdac.phys.gwu.edu/)

**CNS DAC Services**

- The Virginia Tech Partial-Wave Analysis Facility (SAID) has moved to GW!
- New features are being added and will first appear at this site. Suggestions for improvements are always welcomed.
- Once fully operational, this web page will become the main entry for the full range of services presently available through SAID.

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**Instructions for Using the Partial-Wave Analyses**

The programs accessible with the left-hand side navigation bar allow the user to access a number of features available through the SAID program. Contact a member of our group if you are unfamiliar with the SSH version. If you enter choices which are unphysical, you may still get an answer in accordance with the 'garbage in, garbage out' rule. Please report unexpected garbage-out to the management.

**Note:** These programs use HTML forms to run the SAID code. If unfamiliar with the options, run the default setup first. The output is an edited echo of an interactive session which would have resulted had you used the SSH version. If the default example fails to clarify the specific task you have in mind, we can help (just send an e-mail message).

All programs expect energies in MeV units. All of the solutions and potentials have limited ranges of validity. Some are unstable beyond their upper energy limits. Extrapolated results may not make much sense.

**Increments:** The programs will not allow an arbitrary number of points to be generated. As a rule, stay below 100.

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**ACKNOWLEDGMENTS**

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Road Map for Multipoles from GW SAID Analyses of Scattering Data

- $\pi N$ PWA provides the base for Spectroscopy studies for non-strange baryons in all other processes

\[ \pi N \rightarrow \pi N \]
\[ \gamma N \rightarrow \pi N \]
\[ e N \rightarrow e' \pi N \]

$M_R, \Gamma$, and Residue Extraction

Multipole Extraction

$M, E, S$ vs $Q^2$
Summary of Coupled Channel GW SAID Fit of $\pi N$ and $\eta N$ data

[R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C 74, 045205 (2006)]

- $T_{\pi} = 0 - 2600$ MeV          [W = 1080 - 2460 MeV]
- $P_{\pi} = 15 \ [I=1/2] + 15 \ [I=3/2] + 5 \ [\eta N]$
- $Prms = 94 \ [I=1/2] + 80 \ [I=3/2]$
- 4-channel Chew-Mandelstam K-matrix parameterization

$[\pi N, \pi \Delta, \rho N, \eta N]$

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Data</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+ p \rightarrow \pi^+ p$</td>
<td>13344</td>
<td>27242</td>
</tr>
<tr>
<td>$\pi^- p \rightarrow \pi^- p$</td>
<td>11967</td>
<td>22705</td>
</tr>
<tr>
<td>$\pi^- p \rightarrow \pi^0 n$</td>
<td>2933</td>
<td>6091</td>
</tr>
<tr>
<td>$\pi^- p \rightarrow \eta n$</td>
<td>257</td>
<td>628</td>
</tr>
<tr>
<td>DRs</td>
<td>3375</td>
<td>671</td>
</tr>
<tr>
<td>Total</td>
<td>31,876</td>
<td>57,241</td>
</tr>
</tbody>
</table>

- Recent Contribution: HE, CHAOS
- CB, PSI

- In the future, MIPP-FNAL and J-PARC can contribute a lot of hadronic data

- 106 data above 800 MeV
- Very little Pol measurements
Minimization and Normalization Factor $[\chi^2/\text{Data}]$

[R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C 74, 045205 (2006)]

- Modified $\chi^2$ function, to be minimized

$$\chi^2 = \sum_i \left[(X\theta_i - \theta_i^{\text{exp}}) / \varepsilon_i\right]^2 + \left[(X - 1) / \varepsilon_X\right]^2$$

$\theta_i^{\text{exp}}$ measured, $\varepsilon_i$ stat err; $\theta_i$ calculated; $X$ norm const, $\varepsilon_X$ its err

<table>
<thead>
<tr>
<th>Reac</th>
<th>SP06</th>
<th>FA02</th>
<th>KA84</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Norm</td>
<td>UnNorm</td>
<td>Norm</td>
</tr>
<tr>
<td>$\pi^+p$</td>
<td>2.0</td>
<td>6.7</td>
<td>2.1</td>
</tr>
<tr>
<td>$\pi^-p$</td>
<td>1.9</td>
<td>6.2</td>
<td>2.0</td>
</tr>
<tr>
<td>$cxs$</td>
<td>2.1</td>
<td>4.5</td>
<td>2.4</td>
</tr>
<tr>
<td>$\eta n$</td>
<td>2.4</td>
<td>10.1</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Number of datapoints for KA84 corresponds to the modern SAID database.
$\pi N$ Analysis Flow Chart

Intermediate Solution

Calculate DR Constraints

Fit DR + Data

Fit Converges?

$\pi N$ and $\eta N$ Amplitudes

Cook until DONE!
Influence of Spin-rotation Measurements on PWA of Elastic $\pi^N$


$P_{31}$

$A$

$D_{33}$

KA84: Karlsruhe-Helsinki (KH)
KB84: KH Barrelet corrected
SP06: GW DAC fit
Where is Resonance?

[R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C 74, 045205 (2006)]
Here you Are...

• **Main techniques:**
  - Pole on complex energy plane
  - Breit-Wigner fit (data, SES, or global)

• **Additional:**
  - Speed plot, \( Sp(W) \)
  - Argand plot, \( \text{Im}(\text{Re}) \)
  - Crossover energy, \( \text{Re}A = 0 \)
  - Time-delay
  - *etc*
Search for N* and Δ*

- 'Extra' structures?

- Applied directly to the data via BW + bckgr

- Assume: \( S \rightarrow S_RS_B \)

  \[ S_R = 1 + 2iT_R \]

  \[ T_R = \frac{\Gamma_e/2}{[W_R - W - i(\Gamma_e/2 + \Gamma_I/2)]} \]

  \[ \Gamma = \Gamma_e + \Gamma_I \quad \Gamma_e = \rho_e \Gamma R \quad \Gamma_I = \rho_i \Gamma (1 - R) \]

  \[ T_B = K_B(1 - iK_B)^{-1} \quad K_B = a + b(W - W_R) + c \]

- Map \( \chi^2[W_R, \Gamma] \) while searching all other PW prms

  Look for significant improvement
S-waves

L_{(2I)(2J)}

- $L_{(2I)(2J)}$
- S_{11}
- N(2090)*
- $\Delta T - T^* T \leq \text{Im} T$ [unitarity limit]
- $\text{Re} T$
- $\Delta(1900)^{**}$
- $\Delta(2150)^*$
- $S_{31}$

- SP06 vs SES in Ampl
- SP06 vs KA84 in Ampl
- SP06 vs PDG06 in BW

- KA84 [R. Koch, Z Phys C 29, 597 (1985)]
P-waves

\[ P_{11}, N(1710)^{***} \]
\[ N(2100)^* \]
\[ P_{31}, \Delta(1750)^* \]
\[ \Delta(1600)^{***} \]
\[ \Delta(1920)^{***} \]

\[ P_{13}, N(1900)^{**} \]

Amplitude vs. W (MeV)
D-waves

\[ D_{13} \]

\[ D_{33} \]

\[ D_{15} \]

\[ D_{35} \]

\[ N(1700)*** \]

\[ N(1940)* \]

\[ N(2080)** \]

\[ N(2200)** \]

\[ N(2350)* \]

\[ \text{new} \]
F-waves
G-waves

$G_{17}$  

$G_{37}$

$G_{19}$

$G_{39}$

N(2200)*

new
H-waves

$H_{19}$

$H_{39}$

$H_{111}$

$H_{311}$

N(2300)**

new

new
$S_{11}$ within Coupled Channel Fit

[R. Arndt, W. Briscoe, IS, R. Workman, A. Gridnev, Phys Rev C 72, 045202 (2005)]

$N(1535)S_{11}: \Gamma_\eta > \Gamma_\pi$

\begin{itemize}
  \item $N(1535)S_{11}$
  \begin{align*}
  &\text{Soln} & \Gamma_\pi & \Gamma_\eta & \Gamma_{\pi\Delta} & \Gamma_{pN} & \Gamma_\eta / \Gamma \\
  &\text{Fit A} & 30 \pm 2 & 45 \pm 3 & 15 \pm 1 & & 0.50 \\
  &\text{Fit B} & 32 \pm 3 & 45 \pm 4 & 16 \pm 1 & & 0.48 \\
  &\text{Fit C} & 39 \pm 3 & 67 \pm 4 & 9 \pm 2 & & 0.58 \\
  &\text{Fit D} & 42 \pm 6 & 70 \pm 10 & 11 \pm 2 & & 0.57 \\
  \end{align*}

Fit A, C (with Xball)
Fit B, D (no Xball)
**D_{13} within Coupled Channel Fit**

[R. Arndt, W. Briscoe, IS, R. Workman, A. Gridnev, Phys Rev C 72, 045202 (2005)]

- **N(1520)D_{13}:** \( \Gamma_\eta/\Gamma_t = 0.0008 - 0.0016 \)

<table>
<thead>
<tr>
<th>Soln</th>
<th>( \Gamma_\pi ) (MeV)</th>
<th>( \Gamma_\eta ) (MeV)</th>
<th>( \Gamma_{\pi \Delta} ) (MeV)</th>
<th>( \Gamma_{\rho N} ) (MeV)</th>
<th>( \Gamma_\eta /\Gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fit A</td>
<td>68(\pm)1</td>
<td>19(\pm)6</td>
<td>19(\pm)5</td>
<td>19(\pm)5</td>
<td>0.0012</td>
</tr>
<tr>
<td>Fit B</td>
<td>68(\pm)1</td>
<td>19(\pm)6</td>
<td>19(\pm)6</td>
<td>19(\pm)6</td>
<td>0.0016</td>
</tr>
<tr>
<td>Fit C</td>
<td>67(\pm)1</td>
<td>14(\pm)4</td>
<td>24(\pm)4</td>
<td>24(\pm)4</td>
<td>0.0008</td>
</tr>
<tr>
<td>Fit D</td>
<td>67(\pm)1</td>
<td>14(\pm)5</td>
<td>24(\pm)5</td>
<td>24(\pm)5</td>
<td>0.0009</td>
</tr>
</tbody>
</table>

Fit A, C (with Xball)
Fit B, D (no Xball)

- **D_{13} [Mainz (\(\gamma, \eta\))]:** \( \Gamma_\eta /\Gamma = 0.0008 \pm 0.0001 \)
- **D_{13} [Giessen, multi-ch]:** \( \Gamma_\eta /\Gamma = 0.0023 \pm 0.0004 \)
### What is Known about $N^* \to \eta N$


<table>
<thead>
<tr>
<th>PDG</th>
<th>$N^*$</th>
<th>BR$\to\eta N$</th>
<th>SAID</th>
</tr>
</thead>
<tbody>
<tr>
<td>****</td>
<td>N(1520)$D_{13}$</td>
<td>0.0023 ± 0.0004</td>
<td>0.0010 ± 0.0002</td>
</tr>
<tr>
<td>****</td>
<td>N(1535)$S_{11}$</td>
<td>0.52 ± 0.08</td>
<td>0.54 ± 0.04</td>
</tr>
<tr>
<td>****</td>
<td>N(1650)$S_{11}$</td>
<td>0.06 ± 0.03</td>
<td>?</td>
</tr>
<tr>
<td>****</td>
<td>N(1675)$D_{15}$</td>
<td>0.000 ± 0.010</td>
<td>?</td>
</tr>
<tr>
<td>****</td>
<td>N(1680)$F_{15}$</td>
<td>0.000 ± 0.010</td>
<td>?</td>
</tr>
<tr>
<td>***</td>
<td>N(1700)$D_{13}$</td>
<td>0.000 ± 0.010</td>
<td>No</td>
</tr>
<tr>
<td>***</td>
<td>N(1710)$P_{11}$</td>
<td>0.062 ± 0.010</td>
<td>No</td>
</tr>
<tr>
<td>****</td>
<td>N(1720)$P_{13}$</td>
<td>0.040 ± 0.010</td>
<td>?</td>
</tr>
<tr>
<td>**</td>
<td>N(1900)$P_{13}$</td>
<td>0.14 ± 0.05</td>
<td>No</td>
</tr>
<tr>
<td>**</td>
<td>N(1990)$F_{17}$</td>
<td>?</td>
<td>No</td>
</tr>
<tr>
<td>**</td>
<td>N(2000)$F_{15}$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>**</td>
<td>N(2080)$D_{13}$</td>
<td>0.035 ± 0.035</td>
<td>No</td>
</tr>
<tr>
<td>*</td>
<td>N(2090)$S_{11}$</td>
<td>?</td>
<td>No</td>
</tr>
<tr>
<td>*</td>
<td>N(2100)$P_{11}$</td>
<td>0.61 ± 0.60</td>
<td>No</td>
</tr>
<tr>
<td>****</td>
<td>N(2190)$G_{17}$</td>
<td>0.000 ± 0.010</td>
<td>?</td>
</tr>
<tr>
<td>**</td>
<td>N(2200)$D_{15}$</td>
<td>?</td>
<td>No</td>
</tr>
<tr>
<td>****</td>
<td>N(2220)$H_{19}$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>****</td>
<td>N(2250)$G_{19}$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>***</td>
<td>N(2600)$I_{111}$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>**</td>
<td>N(2700)$K_{113}$</td>
<td>?</td>
<td>No</td>
</tr>
</tbody>
</table>

- The attractive fact is that $\pi^- p \to \eta n$ is an `isospin filter' to the nucleon response, as $\eta N$ final states can originate only from the isospin $I = 1/2$ system.
Argand Plots up to 2.5 GeV

[R. Arndt, W. Briscoe, IS, R. Workman, Phys Rev C 74, 045205 (2006)]

- Crosses indicate every 50 MeV step in $W$
- Dots correspond to BW, $W_R = M_R$
- Every PW has a single BW except $S_{11}$ and $F_{15}$ which have two BWs, while $F_{17}$ has no pole
Some History: `new' or well forgotten


Where is \(\Delta(1232)P_{33}\)?

- \(\text{Re}A = 0\) at 'crossover' energy
- But crossover energy is NOT mass

\[
\begin{align*}
\text{BW-fit} \ [+ \text{bgd}] \text{ yields:} \\
M_\Delta &= 1232.86 \pm 0.74 \text{ MeV} \\
\Gamma_\Delta &= 118.06 \pm 1.20 \text{ MeV}
\end{align*}
\]

- \(\text{Pole:} \ W = 1210.6 - i49.7 \text{ MeV}\)
Complex Energy Plane for $S_{11}$ [FA02]

[R. Arndt, W. Briscoe, IS, R. Workman, M. Pavan, Phys Rev C 69, 035213 (2004)]

- **Poles:**
  - $1526 - i 65$ MeV
  - $1653 - i 91$ MeV

- **Branch-points:**
  - $\eta N$ thr: $1487 - i 0$ MeV
  - $\rho N$ thr: $1715 - i 73$ MeV

- **Zero:**
  - $1578 - i 38$ MeV

- **BW's:**
  - $W_R = 1546.7 \pm 2.2$ MeV
  - $\Gamma = 178.0 \pm 12.0$ MeV
  - $W_R = 1651.2 \pm 4.7$ MeV
  - $\Gamma = 130.6 \pm 7.0$ MeV
Complex Energy Plane for $P_{11}$ [SP06]

[R. Arndt, W. Briscoe, S., R. Workman, Phys Rev C 74, 045205 (2006)]

- Pole 1: 1359-i82 MeV
- BW: $W_R = 1485.0 \pm 1.2$ MeV, $\Gamma = 248 \pm 18$ MeV
- Pole 2: 1388-i83 MeV

- Branch-point [$\pi\Delta$ thr] $[1350 - i50$ MeV$]
- Branch-point [$\eta N$ thr] $[1487 - i0$ MeV$]

Two poles at 2 different Riemann sheets, both are very near to the branch-point [$\pi\Delta$-thr], and the additional branch-point [$\eta N$-thr]

A simple BW does not account for such complexity
Direct Measurements of $N(1440)_{P_{11}}$

- **BEPC**: $e^+e^- \rightarrow J/\psi \rightarrow p\pi^-\bar{n} + \bar{p}\pi^+n$
  [M. Ablikim et al. (BES Collaboration), Phys Rev Lett 97 062001 (2006)]

- **SATURNE II**: $\alpha p \rightarrow \alpha' X$
  [H.P. Morsch and P. Zupranski, Phys Rev C 61, 024002 (2000)]

- $M = 1358 \pm 6 \pm 16$ MeV
  $\Gamma = 179 \pm 26 \pm 50$ MeV

- $M = 1390 \pm 20$ MeV
  $\Gamma = 190 \pm 30$ MeV

- Looks similar as pole at 2nd sheet in GW $\pi N$

- $PWA$: $J^P = 1/2^+$
  $M = 1358 \pm 6 \pm 16$ MeV
  $\Gamma = 179 \pm 26 \pm 50$ MeV

- Looks similar as pole at 1st sheet in GW $\pi N$
More Direct Measurements of $N(1440)P_{11}$

- **CELSIUS-WASA**: $pp \rightarrow np\pi^+$
  - [H. Clement et al/nucl-ex/0612015]

- **JLab-RSS**: $ep \rightarrow e'X$
  - [F.R. Wesselmann et al/nucl-ex/0608003]

- $M=1360\text{ MeV}$
  - $\Gamma=140\text{ MeV}$
  - Looks similar as pole at 1st sheet in GW $\pi N$

- $M=1338\pm 10\text{ MeV}$
  - $\Gamma=65\pm 26\text{ MeV}$
  - Looks similar as poles at 1st and 2nd sheets in GW $\pi N$
  - evidence of two poles?

- $M=1346\pm 5\text{ MeV}$
  - $\Gamma=71\pm 35\text{ MeV}$
$P_{11}$ via Argand and Speed plots

- Is standard BW an appropriate form to extract $N(1440)$ from the set of several nearby singularities [2 poles and $\pi\Delta$, $\eta\pi\Delta$, $\eta N$ branch-points with a very prominent cut] ?!!

- $Sp(W) = |dT/dW|$ peak at $W=\text{pole}$ at NonRes$\rightarrow 0$
  $[G.\ Hoehler, \pi N\ Newslett.\ (1993)]$

- Above 1400 MeV, $Sp(W)$ is flat

$W = 1080 \pm 2280$ MeV
### N(1710)P_{11} - What was Known


<table>
<thead>
<tr>
<th>Ref</th>
<th>Mass (MeV)</th>
<th>Width (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KH79</td>
<td>1723±9</td>
<td>120±15</td>
</tr>
<tr>
<td>CMU80</td>
<td>1700±50</td>
<td>90±30</td>
</tr>
<tr>
<td>KSU92</td>
<td>1717±28</td>
<td>480±230</td>
</tr>
<tr>
<td>GW06</td>
<td>not seen</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ref</th>
<th>Re (MeV)</th>
<th>-2xIm (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMU80</td>
<td>1690±20</td>
<td>80±20</td>
</tr>
<tr>
<td>CMU90</td>
<td>1698</td>
<td>88</td>
</tr>
<tr>
<td>KH93</td>
<td>1690</td>
<td>200</td>
</tr>
<tr>
<td>GW06</td>
<td>not seen</td>
<td></td>
</tr>
</tbody>
</table>

• The spread of $\Gamma$, $\Gamma_\pi/\Gamma$, and $\Gamma_\eta/\Gamma$, selected by PDG, is very large
• $\Gamma$ is too large, $\geq 100$ MeV
• If this state is related to the $\Theta^*$ then it would be more natural for the same unitary multiplet (with $\Theta^*$ and $N^*$) to have comparable widths
Direct Anti-Evidences for $N(1710)P_{11}$

- **CLAS single- and double-charged-pion electroproduction off protons data in an isobar approach at $W = 1100$ to $1780$ MeV and $Q^2 = 0.65$ GeV$^2$**
  [I. Aznauryan et al, Phys Rev C 72, 045201 (2005)]:
  'At $Q^2 = 0$, the coupling of the resonance $N(1710)P_{11}$ to $\gamma N$ is small. Our analysis showed that this resonance make minor contribution to the resonant electroproduction cross section'

- **2500 $\gamma p \rightarrow K^+\Lambda$ data below $W = 2500$ MeV in a multipole approach**
  [T. Mart and A. Sulaksono, Phys Rev C 74, 055203 (2006)]:
  'The $3^*$ resonance $N(1710)P_{11}$ that has been used in almost all isobar models within both single-channel and multi-channel approaches is found to be insignificant to the $K^+\Lambda$ photoproduction by both SAPHIR and CLAS data'

- **Combined analysis of CLAS $2\pi$ electroproduction data at photon virtualities from 0.5 to 1.5 GeV$^2$ and for $W$ from 1400 to 1900 MeV**
  [V. Mokeev, PC 2006]:
  'Electroproduction strength $\sqrt{(A_{1/2}^2 + S_{1/2}^2)}$ for $N(1710)P_{11}$ should be below 0.02 GeV$^{-1/2}$'
Narrow Resonances in PWA

• Because PWA (by construction) tends to miss narrow Res with $\Gamma < 30$ MeV
• We assume the existence of a Res and refit over the whole database

• Insertion of a narrow Res in PWA for
  elastic case: $e^{2i\delta} \Rightarrow e^{2i\delta}_R e^{2i\delta}_B$
  $$e^{2i\delta}_R = (M_R - W + i \Gamma_R/2)/(M_R - W - i \Gamma_R/2)$$

  inelastic case: $\eta e^{2i\delta} \Rightarrow \langle a|S|a\rangle = r_a A(W) e^{2i\delta}_R + (1 - r_a) B(W)$
  $$r_a = BR(R \to a) \quad |A(M_R)| = 1 \quad \Sigma r_a = 1$$
  $$\eta \leq 1 \Rightarrow r_a |A(W)| + (1 - r_a) |B(W)| \leq 1$$

• How does this insertion change $\chi^2$? (Will it decrease?)
• $\Delta\chi^2$ due to insertion of a Res into $P_{11}(J^P = 1/2^+)$

- At $|M_R - W| \gg \Gamma_R$, Res contributes $\sim \Gamma_{el}/(M_R - W)$

- Two candidates: $M_R = 1680$ MeV, $1730$ MeV
  $\Gamma_{\pi N} < 0.5$ MeV, $< 0.3$ MeV

- The procedure is less sensitive to $\Gamma_{tot}$
Features

• Refitting
  - Worse description
    ⇒ a Res with corresponding M and Γ is not supported
  - Better description
    ⇒ a Res may exist
    ⇒ effect can be due to various corrections *(eg, thresholds)*
    ⇒ both possibilities can contribute
    Some additional checks are necessary

• A true Res should provide the effect only in a particular PW

• While NonRes source may show similar effects in various PWs
Check other Partial Waves

- \( \Delta \chi^2 \) due to insertion of a Res into \( S_{11} \) (\( J^p = 1/2^- \))

- \( \Delta \chi^2 \) due to insertion of a Res into \( P_{13} \) (\( J^p = 3/2^+ \))

- No effects at \( M = 1680 \) MeV and possible (small) effects at \( M = 1730 \) MeV
Conclusion from Modified $\pi N$ PWA for $S$- and $P$-waves

(dedicated for the search for narrow states, $\Gamma < 30$ MeV)


- 1680 MeV - only one partial wave ($P_{11}$) reveals the effect: support to the resonance, $\Gamma_{\pi N} < 0.5$ MeV
- 1730 MeV - $P_{11}$ may also reveal a resonance with $\Gamma_{\pi N} < 0.3$ MeV but differently: Res is still possible, if accompanied by different corrections

- The Res at 1730 MeV may appear in $P_{13}$ or $S_{11}$ (less probable), if accompanied by different corrections [e.g., thresholds: $N\omega(1720)$, $N\rho(1715)$ ?, $K\Sigma(1685)$]

- The rest of partial waves ($D_{15}$, etc) do not support narrow states
Direct Evidences for $N(1680)P_{11}$ in $\gamma n \rightarrow \eta n$

- **GRAAL**: backward $\gamma n \rightarrow \eta n$
  [V. Kuznetsov, Phys Lett B 647, 23 (2007)]

- **CB-ELSA**: Very preliminary $\sigma(\gamma n \rightarrow \eta n)$

- **LNS**: Very preliminary $\gamma n \rightarrow \eta n$
  [J. Kasagi, YKIS 2006, Nov 2006]

- **Independent CB-ELSA & LNS measurements confirm the GRAAL observation**

- **EtaMAID does not reproduce both $p$ and $n$ data well**

- **There is a good candidate for missed $N^*$**
  - Its width is much less than any $S=0$ $N^*$
  - Agreed with $\gamma n$ vs $\gamma p$ within ChSA

  [M. Polyakov and A. Rathke, Eur Phys J A 18, 691 (2003)]
Complex Plane vs BW fits

- $P_{11}$ has 1 BW and 2 poles
- $P_{31}, D_{35},$ and $G_{39}$, possessed large $W_I$, do not allow well determined on-shell Res prms

**BW fit:**
- $\Gamma$  
- $\Gamma_{\pi N}$

**Poles of amplitudes**
- **Branch points:**
  - $[\pi\Delta \text{ thr}] [1350-i50 \text{ MeV}]$
  - $[\eta N \text{ thr}] [1487-i0 \text{ MeV}]$
  - $[\rho N \text{ thr}] [1715-i73 \text{ MeV}]$
Summary of N* and Δ* finding

- **Standard PWA** reveals only wide Resonances, but not too wide ($\Gamma < 500$ MeV) and possessing not too small BR (BR > 4%)

- **PWA** (by construction) tends to miss narrow Resonances with $\Gamma < 30$ MeV

- **Our study does not** support several N* and Δ* reported by PDG2006:
  - *** $\Delta(1600)P_{33}$, $N(1700)D_{13}$, $N(1710)P_{11}$, $\Delta(1920)P_{33}$
  - ** $N(1900)P_{13}$, $\Delta(1900)S_{31}$, $N(1990)F_{17}$, $\Delta(2000)F_{35}$, $N(2080)D_{13}$, $N(2200)D_{15}$, $\Delta(2300)H_{39}$, $\Delta(2750)I_{313}$
  - * $\Delta(1750)P_{31}$, $\Delta(1940)D_{33}$, $N(2090)S_{11}$, $N(2100)P_{11}$, $\Delta(2150)S_{31}$, $\Delta(2200)G_{37}$, $\Delta(2350)D_{35}$, $\Delta(2390)F_{37}$

- **Our study does** suggest several ‘new’ N* and Δ*:
  - ***** $\Delta(2420)H_{311}$
  - *** $\Delta(1930)D_{35}$, $N(2600)I_{111}$ [no pole]
  - ** $N(2000)F_{15}$, $\Delta(2400)G_{39}$
  - new $N(2245)H_{111}$ [CLAS ?]
\(\pi N \rightarrow \pi \pi N\) in Isobar Model


- 241,214 events for \(\pi N \rightarrow \pi \pi N\) have been analyzed in Isobar-model PWA at \(W = 1320\) to 1930 MeV

- \(\pi N \rightarrow \pi \pi N\) is essential above 1300 MeV, \(\sigma_{2\pi N} \sim \sigma_{\text{inel}}\)

- That is the main source of \(\pi N\) inelastic amplitudes and \(\rho N\) with \(\pi \Delta\) contribution

- This 1984 analysis is rather old and there are no new comprehensive analyses

- Looks promising PWA, \(W = 1274\) to 1370 MeV
  [A.A. Bolokhov, V.A.Kozhevnikov and D.N.Tatarkhin, and S.G.Sherman, Phys Rev C 61, 055203, 1 (2000)]
Recent $\pi N \rightarrow \pi \pi N$ Measurements

- **New data came late**: (most of them are total Xsections):

  $W = 1078$ to $1127$ MeV:
  - $\pi^- p \rightarrow \pi^0 \pi^0 n$ [BNL: J. Lowe *et al*/*Phys Rev C* **44**, 956 (1991)]

  $W = 1221$ to $1356$ MeV:
  - $\pi^+ p \rightarrow \pi^+ \pi^+ n$ [PNPI: A. Kravtsov *et al*/*Nucl Phys B* **134**, 413 (1978)]
  - $\pi^+ p \rightarrow \pi^+ \pi^+ n$ [TRIUMF: M. Sevior *et al*/*Phys Rev Lett* **66**, 2569 (1991)]
  - $\pi^+ p \rightarrow \pi^+ \pi^0 p$ [LAMPF: D. Pocanic *et al*/*Phys Rev Lett* **72**, 1156 (1994)]
  - $\pi^- p \rightarrow \pi^- \pi^- n$ [TRIUMF: M. Kermani *et al*/*Phys Rev C* **58**, 3419 (1998)]
  - $\pi^- p \rightarrow \pi^- \pi^+ n$ [CERN: G. Kernel *et al*/*Z Phys C* **48**, 201 (1990)]
  - $\pi^- p \rightarrow \pi^- \pi^+ n$ [TRIUMF: J. Lange *et al*/*Phys Rev Lett* **80**, 1597 (1998)]

  $W = 1213$ to $1527$ MeV:
  - $\pi^- p \rightarrow \pi^0 \pi^0 n$ [BNL: S. Prakhov *et al*/*Phys Rev C* **69**, 045202 (2004)]

  $W = 1257$ to $1302$ MeV:
  - $\pi^+ p \rightarrow \pi^+ \pi^- n$ [20,000 events] [TRIUMF: M. Kermani *et al*/*Phys Rev C* **58**, 3431 (1998)]

  $W = 1300$ MeV:
  - $\pi^- p \rightarrow \pi^+ \pi^- n$ [PSI: R. Mueller *et al*/*Phys Rev C* **48**, 981 (1993)]

  $W = 2060$ MeV:
  - $\pi^- p \rightarrow \pi^- \pi^+ n$ [40,000 events] [ITEP: I. Alekseev *et al*/*Phys At Nucl* **61**, 174 (1998)]
Backup
Possible Mechanism of $\Theta^+$ Production, $N(2400)$
[Ya. Azimov, IS, Phys Rev C 70, 035210 (2004)]

- **CLAS at JLab:**
  \[ \gamma p \rightarrow \pi^+ n(2400) \rightarrow \pi^+ K^- \Theta^+ \]

- **SPHINX at IHEP:**
  \[ pN \rightarrow Nn(2400) \rightarrow \Sigma^0 K^+ , \eta \]

- **No $\pi N$ PWA has seen an $N(2400)$ at $\pi^- p \rightarrow \pi N$ with $\Gamma_{\text{tot}} \geq 100$ MeV and $\text{BR}(R \rightarrow \alpha) \geq 5\%$ [G. Hoehler, Springer, 1983]

- **V. Kubarovsky et al., PRL 92, 032001 (2004)**

- **L. Landsberg, Phys Rep 320, 223 (1999)**

- **Cut on $M(nK^+)$ in $\Theta^+$ region**

- **Outside $\Theta^+$ region**

- **$N(2400)$?**

- **$\Sigma^0 K^+$**

- **$\eta$**
LE Quantities - $g_{\pi NN}^2/4\pi$

- $\min = 13.756 \pm 0.007$
- $\Delta(g^2/4\pi)$ corresponds to $\Delta \chi^2 = 1$ ['canonical' error]
LE Quantities - $\Sigma$-term

\[ \Sigma = 76.1 \pm 0.5 \text{ MeV} \]

\[ \Sigma = 81 \pm 6 \text{ MeV} \]

[G.E. Hite et al, Phys Rev C 71, 065201 (2005)]